

AN AUTOMATIC PROCEDURE FOR EVALUATION OF MECHANICAL PARAMETERS OF METALLIC MATERIALS

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Abstract

An improved automatic procedure - developed to support the measurement process - for statistical evaluation of Ramberg-Osgood model parameters and relevant uncertainties, based upon higher-order non-linear regression techniques, is described in the paper. A theoretical discussion of the treatment used for Ramberg-Osgood model is presented, with an application to a practical case.

Keywords:

stress-strain curve, metallic material, Young modulus, mechanical parameters.

1 Introduction

Performance evaluation of mechanical components entails knowledge of the relevant material properties. Linear models used with FEM require among other data proper estimates of Young's modulus, while non-linear models require, furthermore, information on plastic properties.

While stress-strain curve provides a major source of information relied upon in the design process, accurate estimation of mechanical parameters is far from straightforward. Statement of uncertainty on values determined, as mandated by quality standards such as ISO 9001:1994 and ISO/IEC/EN 17025:2000, and strongly suggested by the new family of ISO 9000 originated from Vision 2000 project (ISO 9000:2000; ISO 9001:2000; ISO 9004:2000), entail further complication.

Traditional methods for evaluation of Young's modulus (E) from stress-strain data, for instance as proposed by ASTM E 111-82/88 (1988), are typically affected by relative uncertainties of the order of 10%. Evaluation is frequently elusive, since some of the main contribution, due e.g. to pattern irregularities in the initial part of the curve, and to curvature introduced by plasticity onset, are hard to pinpoint. Estimation of E frequently involves a subjective evaluation of the limits of the stress-strain linear zone, with possible introduction of lack of fit if departures from linearity are overlooked, or unnecessary increase of uncertainty should intrinsically linear regions be arbitrarily excluded. Substantial difficulties are also encountered in estimating plasticity characteristics, e.g. strain hardening coefficient.

This paper is mainly aimed at definition of a method for evaluation of Young's modulus, plasticity characteristics and related uncertainties, by analyzing stress-strain data. Effects of initial disturbances are excluded according to an objective procedure, in order to protect results from the effects of subjective, operator dependent decisions. For the stress-strain curve the model of Ramberg-Osgood is used, covering both elastic and plastic deformation ranges. Thus some parameters defining plastic behavior may also be estimated, namely the stress corresponding to a given residual strain and the strain-hardening coefficient.

Neither the Ramberg-Osgood model, nor others for that matter, may however describe accurately the complete strain-stress curve (De Martino *et al.*, 1990). Therefore the proposed method may provide either an accurate description of the elastic range and of the initial part of the plastic deformation range, or an approximate picture of a broader deformation range, with lack of fit and uncertainty increasing the broader the plastic field explored.

The method proposed enables evaluation of uncertainty on all parameters estimated in line with the requirements of the Guide for the Expression of Uncertainty in Measurement (GUM), developed in the framework of BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML (implemented in Italy as UNI CEI 9, now UNI CEI ENV 13005:2000).

An improved automatic procedure for evaluation of mechanical parameters and the relevant uncertainties is described, based upon the higher-order non-linear regression techniques applied to Ramberg-Osgood model.

Non-linear models are widely used in applied statistics (Bates and Watts, 1988); inference on regression coefficients and variance parameters being commonly based on large-sample results and linearisation techniques. However non-linearity of the main function and heteroschedasticity can lead to substantial inaccuracies if the sample size is small or moderate. Higher-order methods enable to overcome this drawback.

2 Stress-strain model

The standard tensile test ranges among the most frequently performed for evaluation of mechanical properties of metals. A standard specimen is gripped between the stationary platen and the moving crosshead of a tensile testing machine, an extensometer with a specified gage length is fastened as required, load is applied at a prescribed rate, and readings of load, longitudinal and possibly lateral extension are logged and stored for further analysis (Bray and Vicentini, 1975).

To obtain the conventional stress-strain curve, nominal stress is plotted against engineering strain, defined as:

$$\sigma_n = \frac{F}{A_0} \quad (1)$$

$$\varepsilon_e = \frac{\Delta l}{l_0} \quad (2)$$

where:

F is the applied load;

A_0 is the original cross-sectional area;

Δl is the extension;

l_0 is the original gage length.

Nominal stress is not however fully representative of actual stress, since specimen's cross-sectional area is affected by load. By the same token, engineering strain is but a biased estimate of actual strain, since actual length is also load dependent.

True stress and strain are related to their nominal and engineering counterparts as follows:

$$\sigma = \sigma_n \cdot (1 + \varepsilon_e) \quad (3)$$

$$\varepsilon = \ln(1 + \varepsilon_e) \quad (4)$$

Since on a relative basis $(\sigma - \sigma_n) / \sigma_n = \varepsilon_e$, errors introduced by the use of nominal stress are small as long as strain is small, typically within elastic range for metals; similar considerations apply to the difference between true and engineering strains. Distortions become substantial for larger strains, that is in the plastic deformation range. A typical true stress – strain curve is shown in Figure 1.

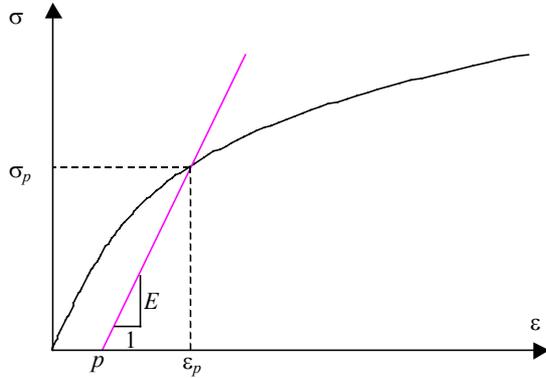


Figure 1: A typical true stress–strain curve.

By and large the curve can be divided into a linear part, typical of the elastic phase, followed by a non-linear part, corresponding to the plastic phase of the deformation process (Colonnetti, 1955).

The curve may be idealized using a dynamic model to describe metal behavior (Mendelson, 1968). Fitting an empirical equation to experimental data may then represent it.

A number of empirical models were proposed in literature (Hetényi, 1950), describing true stress in terms of a function of true strain (with constant parameters for a given material), according to the approach introduced e.g. by Ludwik, Voce and Swift (Johnson and Mellor, 1962):

$$\sigma = \sigma_0 + m \cdot \varepsilon^n \quad (5)$$

$$\sigma = a + (b - a) \cdot (1 - e^{-n \cdot \varepsilon}) \quad (6)$$

$$\sigma = c \cdot (a + \varepsilon)^n \quad (7)$$

True strain may also be written in terms of the sum of elastic and plastic contributions as in a well-known model (Ramberg and Osgood, 1941), as follows:

$$\varepsilon = \frac{\sigma}{E} + p \cdot \left(\frac{\sigma}{\sigma_p} \right)^n \quad (8)$$

where (see Figure 1):

E is the Young's modulus;

p is a given level of plastic strain, typically 0.2%;

σ_p is the stress corresponding to plastic strain p ;

n is a parameter pertaining to transition between elastic and plastic sections of stress-strain curve, and to strain hardening.

By introducing an additional parameter ε_0 such bias as originated at the onset of the loading cycle owing to typical uncertainties occurring there may be taken into account expeditiously. Accordingly, model (8) becomes:

$$\varepsilon = \varepsilon_0 + \frac{\sigma}{E} + p \cdot \left(\frac{\sigma}{\sigma_p} \right)^n \quad (9)$$

3 Parameter estimation

Regression analysis on Ramberg-Osgood's equation may be performed considering E , σ_p and n as unknown parameters, p being as a rule assigned a given value, e.g. 0.2%.

The problem is usually solved by decomposing the fitting procedure into two distinct phases (Rasmussen and Hancock, 1993). In the first linear regression is applied on the first (straight) part of the curve, to estimate Young's modulus. The remaining parameters are then evaluated in terms of the value of E previously obtained. Parameter ε_0 is equated to zero, thus forcing the curve through the origin. According to this methodology, definition of the point separating linear from non-linear phase is crucial.

Another approach for the estimation of Ramberg-Osgood model parameters and the corresponding uncertainties, based on non-linear regression models, is adopted in the present work.

4 Non-linear regression analysis

In the case at hand, a non-linear regression analysis may be performed in order to estimate model parameters and related uncertainties.

4.1 The statistical model

The non-linear statistical model may be defined as follows:

$$\varepsilon_i = \mu(\sigma_i; \theta) + \delta_i \quad \text{with } i = 1, \dots, m \quad (10)$$

where:

ε_i and σ_i are the experimental values of true strain and true stress;

m is the number of measured data;

$\theta = (\varepsilon_0, E, \sigma_p, n)^T$ is the unknown parameters vector;

μ is the mean function, that is the expected value of the true strain ε according to Ramberg-Osgood model;

δ_i are random errors, assumed independent and normally distributed with zero mean and constant variance s^2 ($\delta_i \sim N(0, s^2)$).

4.2 Parameter estimation

Parameter estimation is obtained by maximizing the log-likelihood function of the model (Azzalini, 2001). According to the hypothesis on error distribution introduced above,

$$\ell(\theta) = -\frac{n}{2} \log(s^2) - \frac{1}{2s^2} \sum_{i=1}^n [\varepsilon_i - \mu(\sigma_i; \theta)]^2 \quad (11)$$

The maximum likelihood estimate of s^2 is therefore available in closed form and equal to the residual average sum of squares. On the other hand the parameters vector must be estimated using iterative techniques.

In fact, a maximum likelihood estimate of θ can be obtained solving the likelihood equation:

$$\ell'(\theta) = 0 \quad (12)$$

Using the Newton-Raphson method based on the first order Taylor development of (12), the k -th solution is:

$$\hat{\theta}_k = \hat{\theta}_{k-1} - \left(\frac{d^2}{d\theta d\theta^T} \ell(\hat{\theta}_{k-1}) \right)^{-1} \frac{d}{d\theta} \ell(\hat{\theta}_{k-1}) \quad (13)$$

The algorithm iterates until convergence or until the maximum allowed number of iterations is reached. Stopping rule is written in terms of relative difference between successive estimates, and relative increment of the log-likelihood.

A general method for dealing with near-singularity is provided by the Levenberg-Marquardt compromise: a trust-region strategy in which the length of the step is controlled (Bates and Watts, 1988).

Estimate of inference regions on regression coefficients and variance parameters is commonly based on large samples and linearisation techniques (Draper and Smith, 1998; Seber and Wild, 1989). Non-linearity of the mean function (and heteroschedasticity, if any) may however lead to substantial inaccuracies with small sample size.

In order to overcome these drawbacks higher-order methods may be resorted to, according to the version proposed by Skovgaard (1996) and Severini (1998, 1999). These techniques enable to estimate confidence regions for model parameters.

4.3 Covariance matrix and parameter uncertainties

Implementation of the higher-order method on the statistical model yields an estimate of parameter correlation matrix, and of parameter values.

Furthermore confidence regions with a stated confidence level (usually 95%) for parameters may be

defined by the application of three different statistics, namely Wald, r and r^* , based on the log-likelihood function.

The Wald statistic is obtained as the mean of the second derivative of the log-likelihood function. According to asymptotic theory it follows a χ^2 distribution with a number of degrees of freedom equal to parameter vector dimension. Directed likelihood statistic r , namely the square root of Wald statistic, follows the standard normal distribution.

These statistics are typical of first-order asymptotic theory; approximation to asymptotic distributions may be improved through higher-order asymptotic theory. This leads e.g. to definition of the modified directed likelihood r^* , which is an adjusted version of r .

Confidence intervals derived via application of r^* , obtained using higher-order approximations, should be preferred especially in the case of small sample size (Barndorff-Nielsen and Cox, 1994; Bellio and Brazzale, 1999).

The variability of the maximum likelihood estimates may be assessed by plots of Wald, r and r^* statistics, see Figure 4.

4.4 Initial values

The non-linear regression is dealt with using iterative algorithms, entailing definition of initial values for parameters to be estimated. The problem can be solved in different ways, namely:

- by introducing initial, subjective values, as a competent operator may do according to accumulated experience;
- by exploiting an automatic procedure for approximate statistical evaluation, based, for example, upon the implementation of the so called profile likelihood function method (which is based on a conditioned-likelihood function, in this case, applied to the conditioning parameter n) (Azzalini, 2001; Barndorff-Nielsen and Cox, 1994).

Definition of a set of initial values for unknown parameters is admittedly a delicate task, since picking a wrong set may entail long computing times for the iterative resolution process, or even in a worst case situation to failure to converge altogether.

4.5 Rough data purge

In the initial phase of the tensile test a number of factors such as e.g. faulty alignment, clamping problems, backlash in instruments and machinery, knife edge contact uncertainties, may affect readings up to the point of masking to some extent the physical phenomenon under examination. Sizable deviation between experimental data and expected values may then be observed.

Data purge to exclude misleading data thus generated may be implemented via automatic filtering of irregular initial values, e.g. according to the routine introduced by Bray *et al.* (1996), and followed in the present work. Accordingly, the lower limit of the linear zone is identified by applying an iterative algorithm monitoring the multiple correlation coefficient R pertaining to the linear regression in the elastic phase.

The first step consists in the identification of the n -degree polynomial best approximating experimental data, according to a least-squares regression model. This is required in order to estimate the ordinate of the central point of the linear zone.

The following step is aimed at pinpointing the lower limit of linear range for the purpose at hand. A step p is established (expressed, for example, as a small percent of the full range of σ measured values) and the ordinate of the first tentative lower limit for the linear range is taken by subtracting the value p from central point's ordinate. In the range thus defined, linear regression is performed and R is calculated.

A new, lower ordinate point is then considered by subtracting again the value p from the previously obtained tentative lower limit. Linear regression is performed again; a new value of the multiple correlation coefficient R is calculated and so on. The procedure stops when R is observed to decrease, the last identified ordinate defining the lower limit of the linear range, initial values below such a lower threshold being cut off and excluded from further analysis.

The iterative procedure aimed at identifying the upper limit of the linear range (performed only if need be, e.g. when exclusively linear model fitting is of interest) follows along symmetrical lines. Increment p is now added to central point's ordinate as a first step, and iteratively added as long as no decrease in the value of multiple correlation coefficient R is observed. Deviation from linearity is then due to systematic rather than random effects, curvature being originated by the onset of plastic deformation.

5 Application of the method

The method proposed may be implemented automatically either for on-line data processing in the course of a tensile test or for off-line analysis of collected data. To make full use of the potentiality of the method the following steps should be followed:

- raw data acquisition;
- computation of true stresses and strains;
- data purge through identification of lower limit of linear range, excluding initial irregular data;
- definition of tentative initial values for unknown parameters, either according to previous experience or by automatic procedure, see above;
- non-linear regression for estimation of unknown parameters and related uncertainties;
- analysis of covariance, residual analysis, identification of other significant factors, if any, and evaluation of their effects.

The method outlined was applied to a number of sets of data, covering a comprehensive range of sample sizes and several metals. Satisfactory results were obtained, and comparison with other traditional procedures yielded positive results. An example of application is presented below.

6 Experimental analysis

The tensile test has been performed using a GALDABINI PMA 20 tensile testing system, equipped with model EE/50/1000 extensometer for elongation measurement. Figure 2 shows the experimental load-elongation curve obtained from a standard C40 steel specimen, 10 mm dia. and gage length l_0 of 50 mm.

Elongation and applied load were measured with 1 μm and 1 N instrumental resolutions respectively.

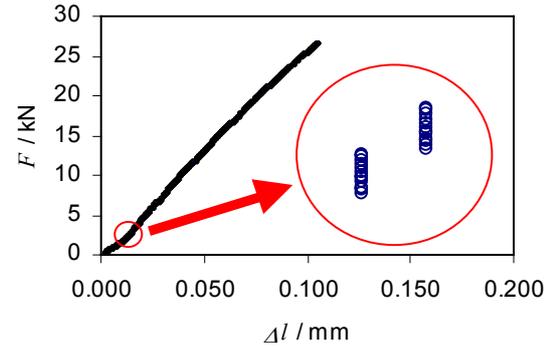


Figure 2: Experimental load-elongation curve obtained in the tensile test. Several load readings were obtained at each recorded elongation.

Over 850 experimental points (couples of load and elongation readings) in the linear phase and in the first part of the plastic phase were logged in during the test.

Force measurement resolution is seen to exceed by far resolution pertaining to elongation measurement. In fact, for a single value of elongation, several values of applied force were recorded, as shown in the enlarged portion of Figure 2.

Exploitation of the load scale near capacity together with use of extensometer with a comparatively short gage length explains a situation not uncommon in tensile testing practice.

Data reduction was performed by averaging out elongation among adjacent readings, and coupling with those average elongation values the grand average of the two sets of load readings involved, thus bringing down the number of load-elongation data points to 103. The corresponding true stress-strain curve is shown in Figure 3.

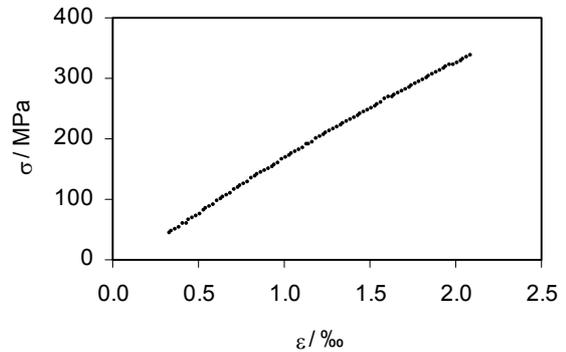


Figure 3: True stress–strain curve obtained from the data shown in Fig. 2, reduced as explained above.

By applying the automated procedure previously described for the determination of Ramberg-Osgood model parameters, the results shown in the following table were obtained.

	$\varepsilon_0 / \%$	E / GPa	σ_p / MPa	n
Value	0.1164	198.7	747.1	2.512
St. Error	0.0027	1.9	8.9	0.095

Table 1: Parameters and corresponding standard errors, estimated according to the method described.

Results shown in Table 1 were obtained from a set of data purged of irregular initial entries according to the method recalled above (Bray et al. 1996); initial values were defined with the profile likelihood function method, and non-linear regression performed.

Graphs shown in Figure 4, representing profiles of r^* , r and Wald statistics, provide further information. Variability of maximum likelihood estimators of the unknown parameters may be thus inferred. It should be underlined that these figures pertain only to Type A uncertainty, since Type B is not dealt with in the present context.

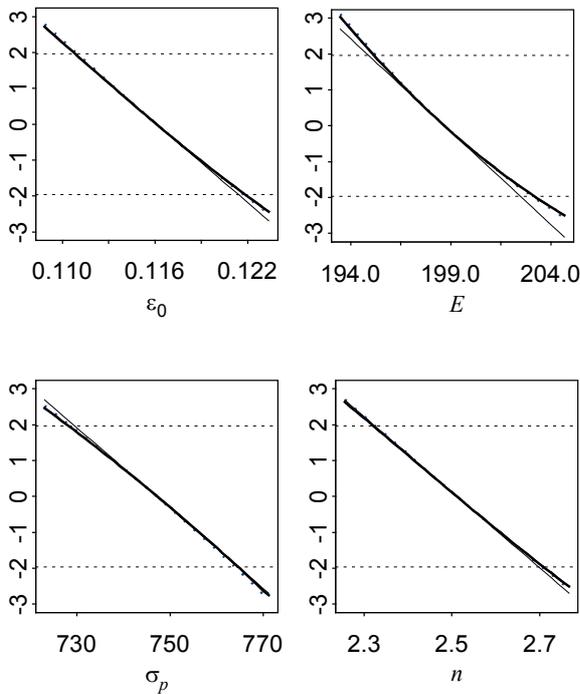


Figure 4: Profile behavior of r^* , r and Wald statistics for the estimated parameters.

Referring to graphs in Figure 4, the horizontal dotted lines enable evaluation of the 95% confidence regions as shown. Bold lines pertain to r^* , dotted lines to r , and thin lines to Wald statistics. Results obtained are shown in Table 2.

Statistic	$\varepsilon_0 / \text{‰}$	E / GPa
r^*	(0.1108; 0.1218)	(195.2; 203.2)
r	(0.1110; 0.1217)	(195.3; 203.0)
Wald	(0.1108; 0.1214)	(194.9; 202.5)
Statistic	σ_p / MPa	n
r^*	(728.3; 764.5)	(2.322; 2.708)
r	(728.7; 763.9)	(2.327; 2.703)
Wald	(729.6; 764.7)	(2.325; 2.698)

Table 2. 95% confidence intervals for parameters obtained with the method proposed.

Taking also into account the rather large number of experimental points, the marginal differences observed among the three statistics considered may be safely disregarded from a practical point of view.

Figure 5 shows the regression curve (calculated using parameters obtained with r^* statistics) together with experimental data.

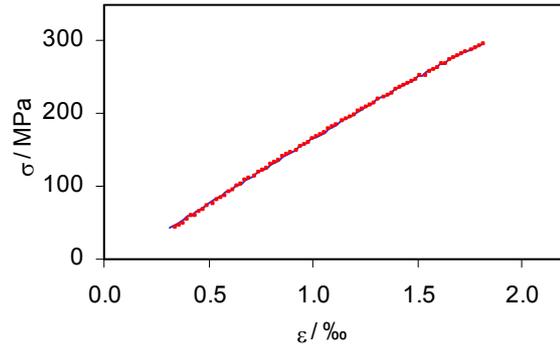


Figure 5: Ramberg-Osgood regression curve and related experimental data.

Residual analysis was performed to find out whether existence of hitherto neglected factors, if any, is hinted at by some peculiar pattern. Figures 6 and 7 show normal probability plot of residuals, and residuals plotted versus experimental true strain.

No substantial evidence may be found to reject the hypothesis of normal distribution of residuals, nor is there any systematic pattern disproving the hypothesis of randomness.

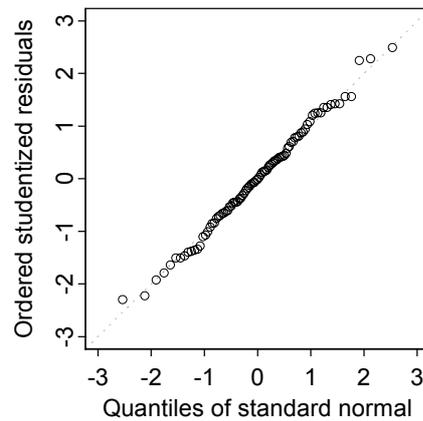


Figure 6: Normal Probability Plot of residuals.

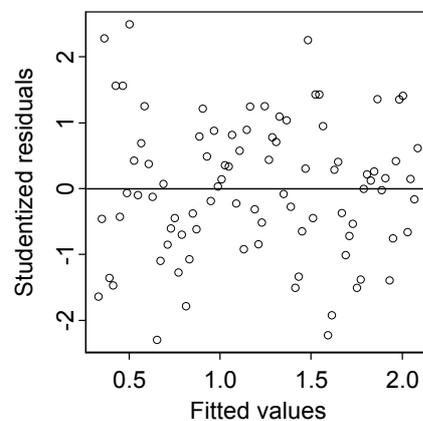


Figure 7: Residuals versus experimental true strain.

7 Conclusions

A procedure is described for automatic determination of mechanical parameters of metallic materials, appearing in the Ramberg-Osgood stress-strain model. The procedure is based upon higher-order non-linear regression methods. Parameters and related uncertainties are estimated as well.

Application of higher-order methods enable to overcome some of the problems associated to small or moderate sample sizes in presence of non-linearity of the mean function and variance heterogeneity.

An improved feature of the procedure is the automatic identification of the initial "gray" zone, over which uncontrolled factors may exert a sizable influence, and the automatic calculation of unknown Ramberg-Osgood parameters together with related uncertainties. Furthermore, the same procedure can easily be applied to other models presenting similar mathematical features.

First results obtained over a number of tests performed on a comprehensive range of specimens appear encouraging, particularly in comparison with estimates obtained via traditional methods. In a nutshell, better estimation of Young's modulus is made possible as information contained in data at onset of plastic deformation and beyond can be exploited in full, as opposed to traditional linear model fitting methods.

Work in process is aimed at further improvements of the automatic method for identification of boundaries of disturbance affected zones and related topics. Several approaches are currently considered, such as segmented polynomials techniques, exclusion principles (e.g. Chauvenet's), multiple criteria decision aiding methods, and indicators of fit based upon residual sum of squares and related statistics.

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